Development and Calibration of a Preliminary Cellular Automata Model for Snow Avalanches

Maria Vittoria Avolio[1], Alessia Errera[2], Valeria Lupiano[3], Paolo Mazzanti[4,5], and Salvatore Di Gregorio[1]

[1] Univ. of Calabria, Dept. of Mathematics & Center of High-Performance Computing, Arcavacata, 87036 Rende (CS), Italy
{avoliomv,dig}@unical.it
[2] eni, exploration & production division, via Emilia 1, 20097 San Donato Milanese, Italy
alessia.errera@eni.com
[3] Univ. of Calabria, Dept. of Earth Sciences, Arcavacata, 87036 Rende (CS), Italy
lupianov@unical.it
[4] NHAZCA S.r.l., spin-off "Sapienza Università di Roma", Via Cori snc, 00177, Roma, Italy
paolo.mazzanti@nhazca.com
[5] Univ. of Rome "Sapienza", Dept. of Earth Sciences, P.le Aldo Moro, 00185, Roma, Italy
paolo.mazzanti@uniroma1.it

Abstract. Numerical modelling is a major challenge in the prevention of risks related to the occurrence of catastrophic phenomena. A Cellular Automata methodology was developed for modelling large scale (extended for kilometres) dangerous surface flows of different nature such as lava flows, pyroclastic flows, debris flows, rock avalanches, etc. This paper presents VALANCA, a first version of a Cellular Automata model, developed for the simulations of dense snow avalanches. VALANCA is largely based on SCIDDICA-SS2, the most advanced model of the SCIDDICA family developed for flow-like landslides. VALANCA adopts several of its innovations: outflows characterized by their mass centre position and explicit velocity. First simulations of real past snow avalanches occurred in Switzerland in 2006 show a satisfying agreement, concerning avalanche path, snow cover erosion depth and deposit thickness and areal distribution.

Keywords: Cellular Automata, Modelling and Simulation, Snow Avalanche.

1 Introduction

Snow avalanches are rapid gravity-driven movements of snow masses down mountain slopes. They may be included in the category of granular flows together with mudflows, debris flows, pyroclastic flows and rock avalanches. In fact, there is experimental evidence for snow avalanches exhibiting all the flow regimes identified in granular flows, from the quasi-static to the collisional, grain-inertia and macroviscous regimes [1].

Dense avalanches have a high density core (100–500 kg/m^3) at the bottom with particle sizes from 1 mm to 1 m, typical flow depths between 0.5 and 5 m and velocities in the range 5–40 m/s. They are a manifestation of the quasi-static and

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collisional regimes. On the other extreme, powder snow avalanches are dilute flows of small snow particles (< 1 mm) suspended in the air by intense turbulence. The density is much lower than in dense avalanches (typically 1–10 kg/m^3), but the flow depth (10–100 m) and average velocity (30–100 m/s) are much larger. In recent years, the important role of the fluidised regime, intermediate between these two end members, has been recognised ([1], [2], [3], [4]). Typical densities and flow velocities are 10–100 kg/m^3 and 30–70 m/s, respectively.

The urgent and increasing need for protection of settlements and traffic routes from snow avalanches has led to several approaches for modelling avalanches over the past 90 years [5].

There is a wide variety of fluid mechanics-based models; they differ in complexity and also with regard to the type of avalanche they describe [5]. Dense snow avalanches can be described by mass-point models, e.g. [6], or continuum models based on the Navier–Stokes or Saint-Venant equations, with a constitutive equation appropriate for flowing snow. In the case of powder snow avalanches and slush flows, it may be necessary to use binary mixture theory to describe the dynamics of the particles and the interstitial fluid satisfactorily [7]. Models of the Saint-Venant type exploit that snow avalanches (and in particular dense avalanches) are shallow flows by integrating the balance equations of mass and momentum (and energy) over the direction perpendicular to the ground ([5], [8] ,[9]) for more details and references to the original works.

A different approach, based on the computational paradigm of Cellular Automata, was adopted by Barpi et al. [10], that developed the model ASCA (cf. Section 2.2) for the simulation of snow avalanches. ASCA simulations of avalanches, that occurred in Susa Valley (Western Italian Alps), were able to reproduce the correct three-dimensional avalanche path and the order of magnitude of the avalanche deposit.

Kronholm et al.[11], instead, used a Cellular Automata based model to show how the spatial structure of shear strength may be critically important for avalanche fracture propagation.

A Cellular Automata (CA), at the basis of the model presented in this work, evolves in a discrete space-time. Space is partitioned in cells of uniform size, each cells embeds a Finite Automaton (FA) computing unit, that changes the cell state according to the states of the neighbour cells, where the neighbourhood conditions are determined by a pattern invariant in time and space [12]. An extension of classical CA [12] was developed in order to model many complex macroscopic fluid-dynamical phenomena, that seem difficult to be modelled in other CA frames (e.g. the lattice Boltzmann method), because they take place on a large space scale.

Such CA can need a large amount of states, that describe properties of the cells (e.g. temperature); such states may be formally represented by means of sub-states, that specify the characteristics to be attributed to the state of the cell and determining the CA evolution. It involves a large amount of states more a complicated transition function, not reducible to a lookup table.

In the case of surface flows, quantities concerning the third dimension, i.e. the height, may be easily included among the CA sub-states (e.g. the altitude), permitting models in two dimensions, working effectively in three dimensions. Furthermore, an algorithm for the minimisation of the differences (in height) [12], [13] was found in this context in order to determine the outflows from a cell toward the remaining cells of its neighbourhood, giving rise to several models for different macroscopic phenomena: lava flows [12], debris/mud flows [12] and rain soil erosion [14].

Explicit velocity solution is adopted: moving flows toward the neighbouring cells are individuated by the sub-states mass, velocity and mass centre co-ordinates. The resulting new mass, mass centre and velocity are computed by composition of all the inflows from the neighbours and the residual quantities inside the cell [15], [16].

This paper illustrates VALANCA (it is the Sicilian word for avalanche and acronym for "Versatile model of Avalanche propagation by LAws and Norms of Cellular Automata"), a new model for the simulation of snow avalanches. VALANCA profits of studies of Barpi et al. [10] but it included new features [13] of SCIDDICA-SS2, ([15], [16]), the most advanced model of the SCIDDICA family for flow-like landslides, developed by some authors of this paper. Some differences, with respect to the ACSA model by Barpi et al., are presented in Section 2.2.

The next section defines the model VALANCA, while the simulation results of two snow avalanches in Davos (Switzerland) are shown in the third section.

2 The Model VALANCA

VALANCA is a two-dimensional CA with hexagonal cells, the state of cell is specified by sub-states, the transition function is constituted by local "elementary" processes, applied sequentially:

$$\text{VALANCA} = < R, X, S, P, \tau >$$

where

- R is the set of regular hexagons covering the region, where the phenomenon evolves.
- X identifies the geometrical pattern of cells, which influence any state change of the central cell: the central cell (index 0) itself and the six adjacent cells (indexes 1,..,6).
- S is the finite set of states of the finite automaton, embedded in the cell; it is equal to the Cartesian product of the sets of the considered sub-states:
  $S_A \times S_D \times S_{TH} \times S_X \times S_Y \times S_{KH} \times S_E^6 \times S_{XE}^6 \times S_{YE}^6 \times S_{KHE}^6 \times S_I^6 \times S_{XI}^6 \times S_{YI}^6 \times S_{KHI}^6$
  - S_A is the cell altitude.
  - S_D is the snow cover depth, that could change into avalanche mass by erosion (Fig.1).
  - S_{TH} is the average thickness of avalanche mass inside the cell (Fig.1), S_X and S_Y are the co-ordinates of the mass centre with reference to the cell centre.
  - S_{KH} is the kinetic head of avalanche mass inside the cell (Fig.1)..
  - S_E is the part of avalanche mass, the so called "external flow", (normalised to a thickness) that penetrates the adjacent cell from central cell, S_{XE} and S_{YE} are the co-ordinates of the external flow mass centre with reference to the adjacent cell centre, S_{KHE} is the kinetic head of avalanche mass flow. There are six components (one for each adjacent cell) for the sub-states S_E, S_{XE}, S_{YE}, S_{KHE}.
  - S_I is the part of avalanche mass toward the adjacent cell, the so called "internal flow", (normalised to a thickness) that remains inside the central cell, S_{XI} and S_{YI} are the co-ordinates of the internal flow mass centre with reference to the central cell centre, S_{KHI} is the kinetic head of avalanche mass flow. There are six components (one for each adjacent cell) for the sub-states S_I, S_{XI}, S_{YI}, S_{KHI}.

- P is the set of the global physical and empirical parameters, which account for the general frame of the model and the physical characteristics of the phenomenon; the next section provides a better explication of the elements of the following set:

$$\{p_a, p_t, p_{fc}, p_{td}, p_{ed}, p_{mt}, p_{pe}\}$$

  - p_a is the cell apothem;
  - p_t is the temporal correspondence of a CA step;
  - p_{fc} is the friction coefficient for avalanche outflows;
  - p_{td}, p_{ed} are parameters for energy dissipation by turbulence and erosion;
  - p_{mt} is the activation thresholds of the snow mobilisation;
  - p_{pe} is the progressive erosion parameters;
- $\tau: S^7 \rightarrow S$ is the deterministic state transition for the cells in *R*. The basic elements of the transition function will be sketched in the next section.

At the beginning of the simulation, we specify the states of the cells in R, defining the initial CA configuration. The initial values of the sub-states are accordingly initialised. In particular, S_A assumes the morphology values; S_D assumes initial values corresponding to the maximum depth of the snow mantle cover except for the detachment area, where the thickness of the detached avalanche mass is subtracted from snowpack depth; S_{TH} is zero everywhere except for the detachment area, where the thickness of detached avalanche mass inside the cell is specified; all values related to the remaining sub-states are zero everywhere.

At each next step, the function τ is applied to all the cells in *R*, so that the configuration changes in time and the evolution of the CA is obtained.

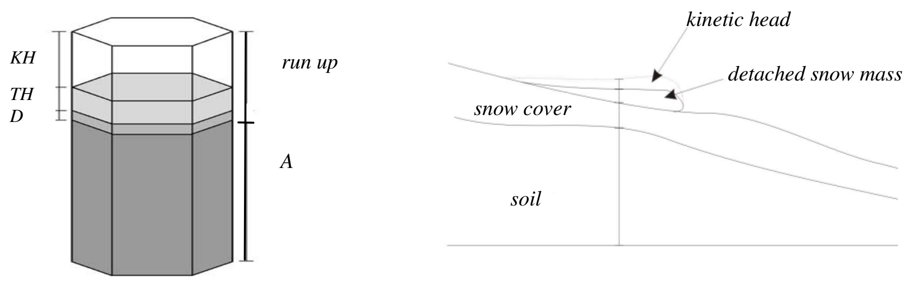

Fig. 1. Left: three-dimensions visualization of the sub-states S_A, S_D, S_{TH} and S_{KH} for a hexagonal cell. Right: an ideal vertical section of a snow flow.

2.1 The VALANCA Transition Function

Four local processes may be considered for VALANCA:

- snow cover, kinetic head and avalanche thickness variation by snow cover mobilisation;
- kinetic head variation by turbulence dissipation;
  avalanche outflows (height, mass centre co-ordinates, kinetic head) determination and their shift deduced by the motion equations;
- composition of avalanche mass inside the cell (remaining avalanche more inflows) and determination of new thickness, mass centre co-ordinates, kinetic head.

In the following, a sketch of the local elementary processes will be given, which is sufficient to capture the mechanisms of the transition function; the execution of an elementary process updates the sub-states. Variables concerning sub-states and parameters are indicated by their abbreviations in the subscripts. When sub-states need the specification of the neighbourhood cell, index is indicated between square brackets. ΔQ means variation of the value of the sub-state S_Q.

Mobilisation Effects. When the kinetic head value overcomes an opportune threshold $KH > mt$ depending on the snow cover features then a mobilisation of the snow cover occurs proportionally to the quantity overcoming the threshold: $pe \cdot (KH - mt) = \Delta TH = -\Delta D$ (the snow cover depth diminishes as the avalanche thickness increases), the kinetic head loss is: $-\Delta KH = ed \cdot (KH - mt)$. The mixing of the eroded snow cover with the earlier avalanche mass involves that the earlier kinetic energy of avalanche mass becomes the kinetic energy of all the avalanche mass, it implicates trivially a further kinetic head reduction.

Turbulence Effect. The effect of the turbulence is modelled by a proportional kinetic head loss at each VALANCA step: $-\Delta KH = td \cdot KH$. This formula involves that a velocity limit is imposed "de facto". A generic case with a maximum value of slope may be always transformed in the worst case of an endless channel with constant maximum value slope. In this case an asymptotic value of kinetic head is implied by infinite formula applications and, therefore, a velocity limit is deduced.

Avalanche Mass Outflows. Outflows computation is performed in two steps: determination of the outflows minimising the "height" differences in the neighbourhood [12] [13] and determination of the shift of the outflows.

The minimisation algorithm defines a central cell quantity d to be distributed, $d = \sum_{i=0}^{n} f[i]$ where $f[i]$ is the flow towards the cell i ($f[0]$ is the part of d, which remains in the central cell); $h[i], 0 \leq i \leq 6$ are the quantities that specify the "height" of the cells in the neighbourhood, to be minimised by contribution of flows: more precisely, the algorithm minimises the expression [16]:

$$\sum_{\{(i,j)|0 \leq i < j \leq 6\}} |(h[i] + f[i]) - (h[j] + f[j])| \qquad (1)$$

Avalanches are rapid flows and imply a run up effect, depending on the kinetic head associated to debris flow. As a consequence, the height minimisation algorithm [17] [18] is applied, considering for the central cell $h[0] = A[0] + KH[0] + D[0]$ and the $d = TH[0]$; $h[i] = A[i] + TH[i] + D[i]$, $1 \leq i \leq 6$ for the adjacent cells; note that $KH[0]$ accounts for the ability of climbing a slope for the flowing avalanche. The minimisation algorithm determines the flows $f[i], 0 \leq i \leq 6$ toward the neighbouring cells ($f[0]$ is the part of d which is not distributed); such flows minimise the expression (1).

The mass centre co-ordinates x and y of moving quantities are the same of all the avalanche mass inside the cell and the form is ideally a "cylinder" tangent the next edge of the hexagonal cell (Fig.2). The height difference $h[0] + d - h[i]$ determines an ideal slope $\theta[i]$ between the two cells 0 and i; a preliminary test is executed in order to account the friction effects, that prevent avalanche outflows, when $\tan\theta[i] < fc$. An ideal length "l" is considered between the avalanche mass centre of central cell and the centre of the adjacent cell i including the slope $\theta[i]$, it represents the maximum allowed path of the outflow.

The $f[i]$ shift "*sh*" is computed for avalanche outflow according to the following simple formula, that averages the movement of all the mass as the mass centre movement of a body on a constant slope with a constant friction coefficient:
$sh = v \cdot t + g \cdot (sin\theta - fc \cdot cos\theta) \cdot t^2/2$, with "g" the gravity acceleration, the initial velocity $v = \sqrt{2\, g \cdot KH}$.

The motion involves three possibilities: (1) only internal flow, the shifted cylinder is completely internal to the central cell; (2) only external flow, all the shifted cylinder is external to the central cell inside the adjacent cell; (3) the shifted cylinder is partially internal to the central cell, partially external to the central cell, the flow is divided between the central and the adjacent cell, forming two cylinders with mass centres corresponding to the mass centres of the internal flow and the external flow.

The kinetic head variation is computed according to the new position of internal and external flows, while the energy dissipation was considered as a turbulence effect in the previous elementary process.

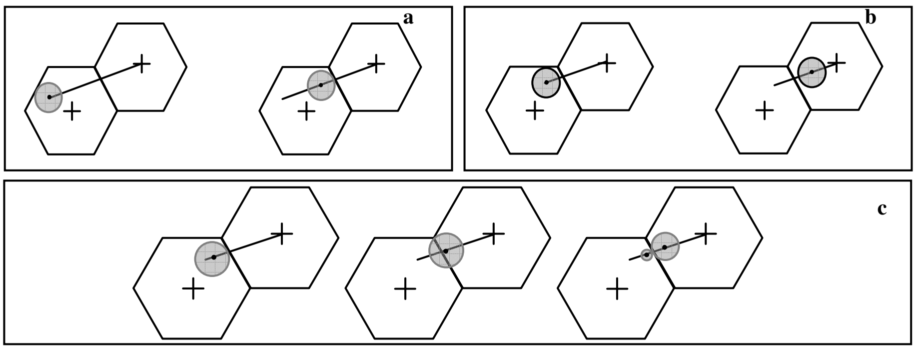

Fig. 2. Determination of the outflow shift. (a) All the cylinder remains in the cell: the flow is only internal and contributes to change the new centre mass of the cell. (b) All the cylinder leaves the cell: the flow is only external. (c) Part of the cylinder crosses the cell: there are both internal and external flows.

Flows Composition. When avalanche mass outflows are computed, the new situation involves that external flows leave the cell, internal flows remain in the cell with different co-ordinates and inflows (trivially derived by the values of external flows of neighbour cells) could exist. The new value of TH is given, considering the balance of inflows and outflows with the remaining snow mass in the cell. A kinetic energy reduction is considered by loss of flows, while an increase is given by inflows: the new value of the kinetic head is deduced from the computed kinetic energy. The co-ordinates determination is calculated as the average weight of X and Y considering the remaining snow mass in the central cell, the internal flows and the inflows.

2.2 Some Relevant Differences between VALANCA and ASCA

ASCA [10] is a CA model for the simulation of snow avalanches, with many analogies with VALANCA. As a matter of fact, both are two-dimensions models based on hexagonal cells and are ruled by the same flow distribution algorithm [11]. The models distinctions are synthesized in the following points.

ASCA shares with many CA models [12] an approach, that doesn't permit to make velocity explicit: a fluid amount moves from a cell to another one in a CA step, which corresponds usually to a constant time. This implies a constant local "velocity" in the CA context of discrete space/time, even if a kind of flow velocity emerges by averaging on the space (i.e. considering clusters of cells) or by averaging on the time (e.g. considering the average velocity of the advancing flow front in a sequence of CA steps). VALANCA, instead, inherits characteristics of the lasts releases of SCIDDICA ([15], [16]), that introduce coordinates of mass centre of flows and computes their shift.. In this case, velocity is locally explicit (cf. Section 2.1). The introduction of mass centers have introduced improvements in simulations in terms of fitness, despite a slight worsening in execution times over the considered simulations.

Note that energy losses related to the kinetic head (cf Section 2.1) are handled in ASCA according different formulae, deduced by approaches of PDE type.

In ASCA, the considered test-case snow avalanche was extremely rapid and thus characterised by relevant run-up effects, whose physical meaning is the minimum height of an obstacle needed to stop the motion of a mass with thickness moving at a certain velocity. Here, the run-up is determined by the thickness of the snow plus a fictitious height, which corresponds to the kinetic head and represents (Fig.1), in the minimization process, a conservative quantity which has to be distributed among the neighboring cells in order to reach the conditions of maximum stability. At the contrary, in the VALANCA model, the run-up effect for fast moving snow avalanches is expressed in a different manner. Here, the kinetic head is not considered as a whole with the snow (i.e., it is not considered as a mobile part during the minimization process) but computed separately from it in order to explicitly consider the physical characteristics related to energy loss and avalanche velocity related to the kinetic head itself (cf. Section 2.1).

3 VALANCA Applications to Real Cases of Snow Avalanches

VALANCA was developed in ANSI C++ in order to obtain both a well structured and extensible source code. The program is characterised by a command line interface that allows the user to interactively control all input/output and simulation, in order that the user can to visualize the simulation in real time. Through the viewer module, it is also possible to observe the DTM over which the phenomenon evolves and perform both a visual and quantitative comparison with the real case in terms of the fitness function f_a, later defined. A first validation and calibration of the parameters of VALANCA have been performed by back-simulating two snow avalanches in Davos (Switzerland) occurred in 2006 and well described in Errera [19]. The same set of parameters have permitted to reproduce the two considered snow avalanches with a great level of accuracy.

First calibrations were performed as usual by a preliminary trial and error method; results were enough good that it was not necessary to use our automatic "long time" calibration techniques [20] (e.g. by means of Genetic Algorithms).

Simulation times depend on the number of active cells processed for each step and on the number of steps, necessary to complete the phenomenon: 10000 cells in a step last approximately 0.5 s for a 2.4GHz dual-core PC. Gotschnawang takes about three minutes with a 199 x 283 matrix excluding interactive graphical output.

During winter season the Davos area is affected by a big number of events. Furthermore, test avalanches were selected since they were well known in terms of areal path, thickness, deposit, velocity during the propagation etc.

The first event analysed (Gotschnawang) is quite challenging since it occurred in an open slope, while the second one (Rüchitobel) represents an interesting example of channelled snow avalanche. Detailed data of snow avalanches were available, among them the release area and volume, avalanche path, the spatial distribution and local thickness of the final deposit, the snow density in the snow cover and in the deposit. Furthermore, in a few cases the propagation velocities could be estimated at specific points. Snow cover entrainment occur in both cases, and also traces of fluidized flow were detected. However, the model simulates, as first attempt, only dense flows.

Both events were simulated by using a 5 *m* cell-size DTM of the area derived from aereophotogrammetry. Several simulations were performed in order to calibrate the parameters and best results are described in what follows.

Fig. 3. Gotschnawang avalanche. Outline of the 2006-01-20 Gotschnawang avalanche (Davos, Switzerland). The extent of the fracture line is indicated by the dotted line.

A first comparison between the real events and the simulated ones is performed by a fitness function f_a [20] concerning areas and computed by the following formula $\sqrt{(R \cap S)/(R \cup S)}$, where R is the set of cells affected by the avalanche in the real event and S the set of cells affected by the avalanche in the simulation. It returns a normalised value between 0 (complete failure) and 1 (perfect simulation). Further comparisons for good values of $f_a(>0.7)$ are performed on erosion and deposits.

3.1 The Gotschnawang Snow Avalanche

On January 20, 2006 a dry-snow avalanche was released artificially from the Gotschnawang slope in the Parsenn ski area (municipality of Klosters, eastern Switzerland) to protect the intermediate station of the Gotschna telepherique. The avalanche covered an area of 230 ha and it was 750 m long (projected length) with a vertical drop of 460 m. This amounts to a runout angle of 31.5°, which is a fairly large value for a dry-snow avalanche of this size. The site is an open but very hummocky slope with a straight track. The release area was 400 m wide, and we estimated the release depth at 40 cm (Fig.3).

The avalanche developed a fluidized layer whose deposits are visible both on the right-hand side and in the frontal part. The fluidized layer ran up to 50 m farther than the dense part and its mass was estimated at 100 tons, compared to 3000 tons for the dense part.

The snow avalanche was back-analysed by the VALANCA model by using a 5 *m* cells DTM by taking into account the release area, the portion of the slope covered by the snow mass and the erosion during the propagation.

A surprising areal agreement between the real event and the simulation was achieved (Fig.4) corresponding to a fitness f_a= 0.92.

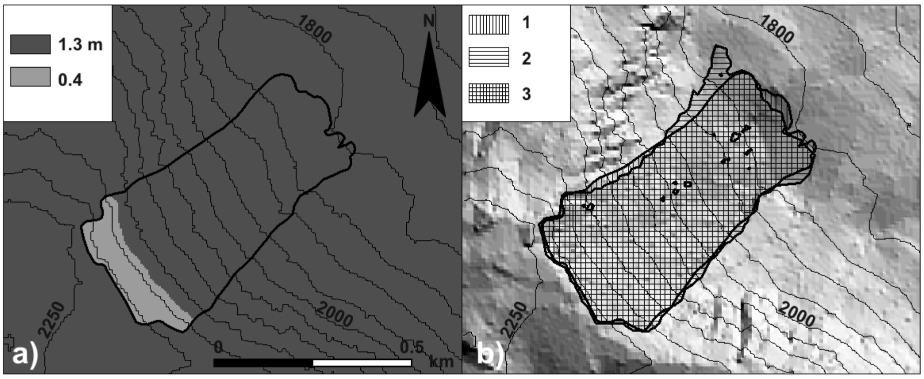

Fig. 4. The 2006 avalanche in Gotschnawang: (a) snow cover with detachment area and real event, (b) post-event DEM, superposition (3) of real (1) and simulated (2) event.

3.2 The Rüchitobel Snow Avalanche

On January 18, 2006 a dry-snow avalanche occurred in the Rüchitobel gully, Dischma Valley (Davos, eastern Switzerland) (Fig.5). The avalanche covered an area of 10 ha and it was 1167 m long (projected length) with a vertical drop of 630 m. The irregularly shaped release area was 195–290 m wide, with an estimated average release depth of 90 cm. For about 650 m (projected length), the flow was channelled in a winding gully whose bottom is around 10–15 m wide and was covered by 0.5–1.5 m deep deposits from this avalanche, stacked over those from earlier ones, while the new snow was eroded completely.

On the upper parts of the gully banks and to the sides, the traces of fluidised layer were clearly observable to a height of 10–15 m above the gully bottom in that the snow was completely eroded away, without any deposits. In the most pronounced bend, the angle between the top flow marks of the fluidised part on either side and the tilt of the surface of the dense deposit indicated maximum flow velocities 28–38 m/s and 10–20 m/s, respectively.

Fig. 5. Rüchitobel avalanche. Outline of the 2006-01-18 Rüchitobel avalanche (Davos, Switzerland). The extent of the fracture line is indicated by the top dotted line.

On the left-hand side and also at the distal end of the runout area, the deposit showed the characteristic features expected from fluidised flow. The mass of the dense deposit was estimated at 4000 *tons* while the fluidised one was only 40–50 *tons* (1% of the avalanche mass).

The snow avalanche was back-analysed by the VALANCA model by using a 5 *m* cells DTM by taking into account the release area, the portion of the slope covered by the snow mass and the erosion during the propagation. The best simulation results corresponded to a value of f_a close to 0.81 which is considered a satisfying preliminary result if the complex geometry of the avalanche path is taken into account (Fig.6).

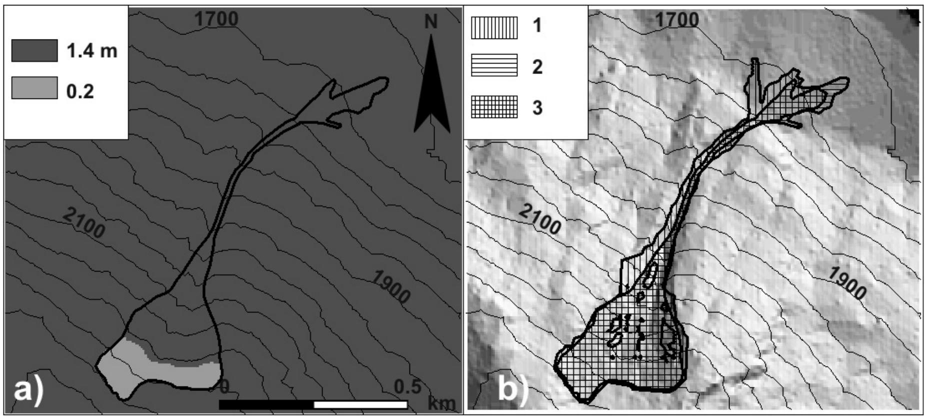

Fig. 6. The 2006 avalanche in Rüchitobel: (a) snow cover with detachment area and real event, (b) post-event DEM, superposition (3) of real (1) and simulated (2) event

4 Conclusions

The CA model VALANCA has been developed which is suitable for the simulation of snow avalanche dynamics. Preliminary validation and calibration of the model have been performed by back-analysing the Rüchitobel and Gotschnawang 2006 snow avalanches. Preliminary results, discussed in this paper, prove the ability of the model to simulate such a type of events in a satisfying way. The real path of the snow avalanche has been well simulated in both open and channelled slopes. However in spite of the encouraging results several improvements (mainly in the numerical management of the erosion and snow entrainment and in the avalanche velocity) are still needed in order to use such a model for forecasting analyses of snow avalanches propagation and their interaction with structures and human settlements.

Furthermore, we are confident that VALANCA could be usefully used in hazard analyses for snow avalanches. With this aim, applications to other cases of different type of snow avalanches have been already planned.

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